Answers for Lesson 12-4, pp. 691–692 Exercises

1. 46

2. 50

3. $x = 60; y = 70$

4. 60

5. $x = 115; y = 74$

6. $x = 108; y = 72$

7. 160°

8. 125

9. 15

10. 11.5

11. 13.2

12. 3.5

13. $x = 25.8; y = 12.4$

14. $x = 5.3; y = 2.9$

15. about 270.8 ft

16. It could range from 256.9 ft to 285.2 ft.

17. $360 - x$

18. $180 - x$

19. $180 - y$

20. 26.7

21. 16.7

22. 14.1

23. $x = 10.7; y = 10$

24. $x = 8.9; y = 2$

25. $x = 10.9; y = 2.3$

26. You must use $(7.5 + 6)6$ or the entire segment length.

27. 95, 104, 86, 75

28. **a.** $30; 30 < m\angle Y < 180; 0 < m\angle Z < 30$

b. If the $\angle$ measure is ≤ 30, the ship is safe.

29. Answers may vary. Sample: Since they are inscribed $\measuredangle$s: $m\angle BED = \frac{1}{2}m\overset{\frown}{BD}$ and $m\angle ABE = \frac{1}{2}m\overset{\frown}{AE}$. Apply the Ext. $\angle$ Thm. to $\triangle BCE$ to prove that $m\angle C = \frac{1}{2}(m\overset{\frown}{AE} - m\overset{\frown}{BD})$.

30. Given: $\overline{AB}$ tangent to $\odot O$ at A, $\overline{BC}$ tangent to $\odot O$ at C; Prove: $m\angle B = \frac{1}{2}(360 - 2m\overset{\frown}{AC})$.

1. Construct $\overline{AC}$.

2. $m\angle A = \frac{1}{2}m\overset{\frown}{AC}$ (The measure of an $\angle$ formed by a tangent and a chord is half the measure of the intercepted arc.)

3. $m\angle C = \frac{1}{2}m\overset{\frown}{AC}$ (The measure of an $\angle$ formed by a tangent and a chord is half the measure of the intercepted arc.)

4. $m\angle B = 180 - m\angle A - m\angle C$ ($\triangle$ $\angle$ Sum Thm.)

5. $m\angle B = 180 - \frac{1}{2}m\overset{\frown}{AC} - \frac{1}{2}m\overset{\frown}{AC}$ (Subst.)

6. $m\angle B = 180 - m\overset{\frown}{AC}$

7. $m\angle B = \frac{1}{2}(360 - 2m\overset{\frown}{AC})$ (Distr. Prop.)

Given: $\overline{BC}$ secant and $\overline{AB}$ tangent to $\odot O$. $\overline{AB}$, $\overline{BC}$ intersect at B and $\overline{AB}$ tangent to $\odot O$ at A. $\overline{BC}$ intersects $\odot O$ at D; Prove: $m\angle B = \frac{1}{2}(m\overset{\frown}{AC} - m\overset{\frown}{DA})$.

1. Construct $\overline{AD}$.

2. $m\angle A = \frac{1}{2}m\overset{\frown}{AD}$ (The measure of an $\angle$ formed by a tangent and a chord is half the measure of the intercepted arc.)

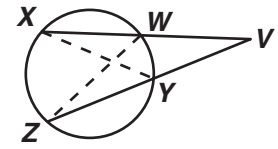
3. $m\angle ADC = \frac{1}{2}m\overset{\frown}{AC}$ (The measure of an inscribed $\angle$ is half the measure of its intercepted arc.)

4. $m\angle B = m\angle ADC - m\angle BAD$ (Subtr. and Ext. $\angle$ Thm.)

5. $m\angle B = \frac{1}{2}m\overset{\frown}{AC} - \frac{1}{2}m\overset{\frown}{AD}$ (Subst.)

6. $m\angle B = \frac{1}{2}(m\overset{\frown}{AC} - m\overset{\frown}{AD})$ (Distr. Prop.)

31. Given: A circle with secant segments $\overline{XV}$ and $\overline{ZV}$; Prove: $XV \cdot WV = ZV \cdot YV$.



1. Construct $\overline{XY}$ and $\overline{ZW}$.

2. $\angle XVY \cong \angle ZVW$ (Reflexive Prop. of $\cong$)

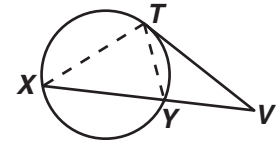
3. $\angle VXY \cong \angle WZV$ (2 inscribed $\measuredangle$s that intercept the same arc are $\cong$.)

4. $\triangle XVY \sim \triangle ZVW$ (AA$\sim$)

5. $\frac{XV}{ZV} = \frac{YV}{WV}$ (In similar figures, corr. sides are proport.)

6. $XV \cdot WV = YV \cdot ZV$ (Mult. Prop.)

32. Given: A circle with tangent $\overline{TV}$ and secant $\overline{XV}$; Prove: $XV \cdot YV = (TV)^2$.



1. Construct $\overline{TX}$ and $\overline{TY}$.

2. $m\angle TXV = \frac{1}{2}m\overset{\frown}{TY}$ (The measure of an inscribed $\angle$ is half the measure of the intercepted arc.)

3. $m\angle VTY = \frac{1}{2}m\overset{\frown}{TY}$ (The measure of an $\angle$ formed by a chord and a tangent is half the measure of the intercepted arc.)

4. $m\angle TXV = m\angle VTY$ (Trans. Prop. of =)

5. $\angle TVY \cong \angle TVX$ (Reflexive Prop. of $\cong$)

6. $\triangle TVY \sim \triangle XVT$ (AA$\sim$)

7. $\frac{YV}{TV} = \frac{TV}{XV}$ (In similar figures, corr. sides are proport.)

8. $XV \cdot YV = TV^2$ (Mult. Prop.)

33. Answers may vary. Sample: If the given pt. is on the circle, then a line through the given pt. can only intersect the circle tangentially or one other place. It follows that one segment has length zero, so the product of the segments is always zero.

34. **1.** $m\angle 1 = \frac{1}{2}m\overset{\frown}{QRP} - \frac{1}{2}m\overset{\frown}{PQ}$ ($\frac{1}{2}$ diff. of intercepted arcs)

2. $m\angle 1 + m\overset{\frown}{PQ} = \frac{1}{2}m\overset{\frown}{QRP} + \frac{1}{2}m\overset{\frown}{PQ}$ (Add. Prop. of = and Distr. Prop.)

3. $m\angle 1 + m\overset{\frown}{PQ} = \frac{1}{2}(m\overset{\frown}{QRP} + m\overset{\frown}{PQ})$ (Distr. Prop.)

4. $m\angle 1 + m\overset{\frown}{PQ} = \frac{1}{2}(360)$ (circle has 360°)

5. $m\angle 1 + m\overset{\frown}{PQ} = 180$ (Subst.)

35. **1.** $m\angle 1 = \frac{1}{2}m\overset{\frown}{QRP} - \frac{1}{2}m\overset{\frown}{PQ}$ $\left(\frac{1}{2}\right.$ diff. of intercepted arcs$\left.\right)$

2. $m\angle 2 = \frac{1}{2}m\overset{\frown}{RQP} - \frac{1}{2}m\overset{\frown}{RP}$ $\left(\frac{1}{2}\right.$ diff. of intercepted arcs$\left.\right)$

3. $m\angle 1 + m\angle 2 = \frac{1}{2}m\overset{\frown}{QRP} + \frac{1}{2}m\overset{\frown}{RQP} - \frac{1}{2}m\overset{\frown}{PQ} - \frac{1}{2}m\overset{\frown}{RP}$ (Subst.)

4. $m\angle 1 + m\angle 2 = \frac{1}{2}m\overset{\frown}{QR} + \frac{1}{2}m\overset{\frown}{RP} + \frac{1}{2}m\overset{\frown}{QR} + \frac{1}{2}m\overset{\frown}{PQ} - \frac{1}{2}m\overset{\frown}{PQ} - \frac{1}{2}m\overset{\frown}{RP}$ (Arc. Add. Post. and Distr. Prop.)

5. $m\angle 1 + m\angle 2 = m\overset{\frown}{QR}$ (Distr. Prop.)

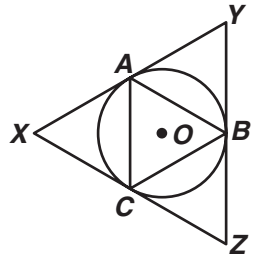
36. **1.** $(PQ)^2 = (QS)(QR)$ (Prod. of segments is const.)

2. $b^2 = (c + a)(c - a)$ (Subst.)

3. $b^2 = c^2 - a^2$ (Distr. Prop.)

4. $b^2 + a^2 = c^2$ (Add. Prop. of =)

37.



$m\overset{\frown}{AB} = m\overset{\frown}{BC} = m\overset{\frown}{AC} = 120$, since chords $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are all $\cong$. So the measures of $\angle X$, $\angle Y$, and $\angle Z$ are $\frac{1}{2}(240 - 120) = 60$, and $\triangle XYZ$ is equilateral.