Algebra 1 Notes

- Graph $y = ax^2 + bx + c$

How does the graph of $y = \frac{1}{2}x^2 - 2x + 5$ differ from the graph of $y = x^2$?

x	0	1	2	3	4
y					

- wider
- opens up
- moves to right
- vertex = (2,3)
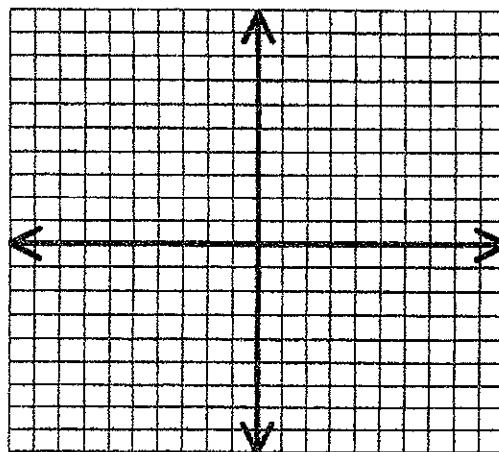
| The graph $y = ax^2 + bx + c$ is a parabola that:	
opens up if a is positive	opens down if a is negative
is narrower than the graph of $y = x^2$ if $\|a\| > 1$	is wider than the graph of $y = x^2$ if $\|a\| < 1$
has an axis of symmetry of $x = \frac{-b}{2a}$	has a vertex with an x-coordinate of $x = \frac{-b}{2a}$
has a y-intercept of $(0, c)$	

EXAMPLES

1. $y = -x^2 - 4x + 6$

opens: _____ AOS $x = -2$

wider, narrower, or neither?

vertex $(-2, 10)$ y-int $(0, 6)$

x					
y					
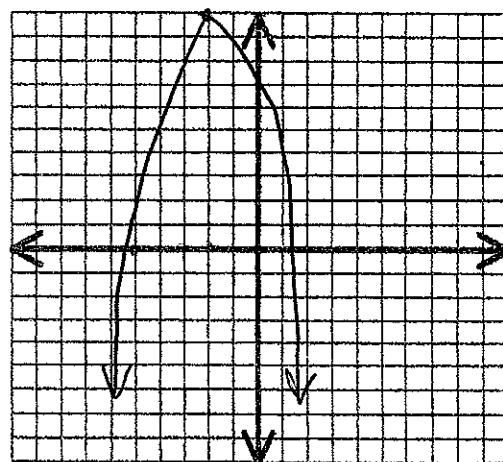
2. $y = -2x^2 - 12x - 8$

opens: _____ AOS $X = 3$

wider narrower neither?

vertex $(-3, 10)$ y-int $(0, -8)$
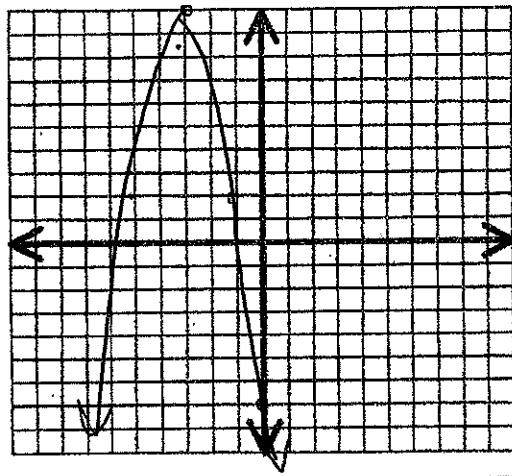
x					
y					

3. $y = \frac{1}{4}x^2 - x + 2$

opens _____ AOS $X = 2$

wider narrower neither?

vertex $(2, 1)$ y-int $(0, 2$
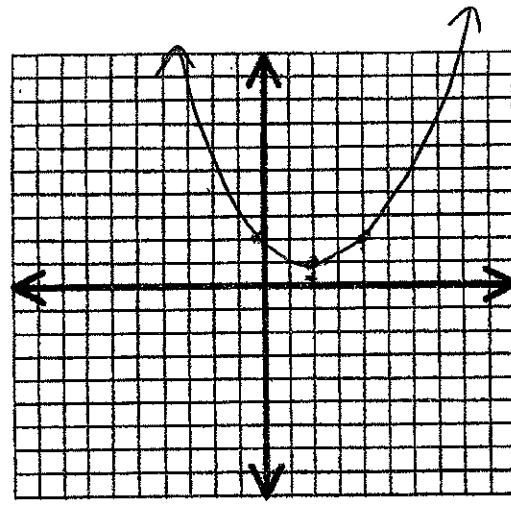
x					
y					

Algebra 1 Notes

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Use Square Roots to Solve Quadratic Equations

$\sqrt{\ \#\ }$	$\sqrt{\ }$	$-\sqrt{\ }$	$\pm\sqrt{\ }$

A Radical Expression is in simplest form when:

1. No radicand has a perfect square factor other than 1.
2. The radicand contains no fractions.
3. No radicals appear in the denominator of a fraction. (You will explore this more in Algebra 2)

Examples of Rule #1: No radicand has a perfect square factor other than 1.

1. $\sqrt{9}$ 3	2. $-\sqrt{49}$ -7	3. $\sqrt{100}$ 10	4. $\pm 2\sqrt{16}$ ± 8
5. $\sqrt{12}$ $2\sqrt{3}$	6. $-\sqrt{8}$ $-2\sqrt{2}$	7. $\sqrt{192}$ $8\sqrt{3}$	8. $\pm\sqrt{27}$ $\pm 3\sqrt{3}$

Examples of Rule #2: The radicand contains no fractions.

9. $\sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{2}$	10. $\pm\sqrt{\dfrac{16}{81}} = \pm\dfrac{4}{9}$

Examples of Rule #3: No radicals appear in the denominator of a fraction.

11. $\dfrac{5}{\sqrt{9}} = \dfrac{5}{3}$	12. $\dfrac{12}{-\sqrt{36}} = -2$

Special cases: Is $-\sqrt{25}$ the same as $\sqrt{-25}$?   NO

You can use square roots to solve quadratic equations in the form of $ax^2 + c = 0$.

A. Isolate the quadratic term x^2 to one side of the equation.
B. Take the square root of both sides to solve. **REMEMBER** to include both roots!!!
C. **Exact solutions** leave irrational #s in simplest radical form.
D. Use a calculator to find **approximate solutions** (round to the nearest hundredth).

Solve the following equations. List answers as **both exact and approximate.**

13. $2x^2 = 40$	14. $n^2 - 18 = -18$	15. $b^2 + 12 = -7$
$x = \pm 2\sqrt{5}$ $= \pm 4.47$	$n = 0$	No real solutions
16. $4x^2 = 9$	17. $4x^2 - 10 = 98$	18. $3x^2 - 35 = 45 - 2x^2$
$x = \pm \dfrac{3}{2}$ $= \pm 1.5$	$x = \pm 3\sqrt{3}$ $= \pm 5.20$	$x = \pm 4$

Algebra 1 Notes   Name_____

Solve Quadratic Equations by the Quadratic Formula

The Quadratic Formula
$$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Used to solve ANY quadratic equation in the form of $ax^2 + bx + c = 0$.

1. Make sure the equation is set = 0!!
2. Identify the values for a, b, and c.
3. Substitute these values into the formula. Be very careful of signs and order of operations!!

Solve the following quadratic equations using the Quadratic Formula.

1. $3x^2 + 5x = 8$ $X = \dfrac{-8}{3}, 1$	2. $2x^2 - 4 = 3x$ $X = 2.35, -.85$
3. $x^2 + 4x + 1 = 0$ $X = -.27, -3.73$	4. $3x^2 = 12$ $X = 2, -2$

Homework:

Solve the following quadratic equations using the Quadratic Formula. Give BOTH exact and estimated values.

1. $x^2 - 5x = 14$ $X = -2, 7$	2. $2x^2 + 9x + 7 = 3$ $X = -\frac{1}{2}, -4$
3. $x^2 + 10x + 1 = 0$ $X = -.10, -9.90$	4. $5x^2 - 3 = -2x$ $X = -1, 3/5$
5. $-2x^2 + 32 = 0$ $X = 4, -4$	6. $4x^2 - 12x + 9 = 0$ $X = \frac{3}{2}$

ZERO PRODUCT PROPERTY:	If $a = 0$ OR $b = 0$ then $ab =$ _____

a and b are known as _____.

The <u>zero product property</u> is used to solve an equation when one side is 0 and the other side is a product of polynomials (factors). The solutions are called _____.

7. $(x+5)(x-3) = 0$	8. $(x-4)(2x-6) = 0$	9. $x(2x-7) = 0$

To factor and use the zero product property to solve polynomial equations, remember these steps:

1. Set each equation = 0;
2. Factor the polynomial (start by looking for a GCF);
3. Set each factor = 0;
4. Solve each equation;
5. Check your roots in the original equation.

10. $a^2 + 5a = 0$	11. $3x^2 = 81x$	12. $12y^2 = 48y$
$a = -5, 0$	$x = 0, 27$	$y = 0, 4$
13. $(4x-2)(2x+3) = 0$	14. $14x = 21x^2$	15. $(x+6)(x-6) = 0$
$x = \dfrac{-3}{2}, \dfrac{1}{2}$	$x = 0, \dfrac{2}{3}$	$x = -6, 6$

Name_____

Solving Quadratic Equations by Factoring

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Solve each equation by factoring. odds only

1) $(k+1)(k-5)=0$

$K = -1, 5$

2) $(a+1)(a+2)=0$

3) $(4k+5)(k+1)=0$

$K = \dfrac{-5}{4}, -1$

4) $(2m+3)(4m+3)=0$

5) $x^2 - 11x + 19 = -5$

$X = 3, 8$

6) $n^2 + 7n + 15 = 5$

7) $n^2 - 10n + 22 = -2$

$n = 6, 4$

8) $n^2 + 3n - 12 = 6$

9) $6n^2 - 18n - 18 = 6$

$n = 4, -1$

10) $7r^2 - 14r = -7$

11) $n^2 + 8n = -15$

$n = -5, -3$

12) $5r^2 - 44r + 120 = -30 + 11r$

13) $-4k^2 - 8k - 3 = -3 - 5k^2$

$K = 8, 0$

14) $b^2 + 5b - 35 = 3b$

15) $3r^2 - 16r - 7 = 5$

$r = -\dfrac{2}{3}, 6$

16) $6b^2 - 13b + 3 = -3$

17) $7k^2 - 6k + 3 = 3$

$K = \dfrac{6}{7}, 0$

18) $35k^2 - 22k + 7 = 4$

19) $7x^2 + 2x = 0$

$X = -\dfrac{2}{7}, 0$

20) $10b^2 = 27b - 18$

21) $8x^2 + 21 = -59x$

$X = -\dfrac{3}{8}, -7$

22) $15a^2 - 3a = 3 - 7a$

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Factoring Differences of Perfect Squares and Factoring Completely

1. Multiply: $(2x-3)(2x+3)$	2. Multiply: $(4x+5)(4x-5)$

What type of polynomial are these?

Which term is missing when written in descending order?

What is special about each term?

What operation is occurring?

To factor the **difference of perfect squares**: $a^2 - b^2$ follow the pattern $(\quad\quad)(\quad\quad)$

Examples

3. $25x^2 - 16$ $(5x+4)(5x-4)$	4. $81x^2 - 4$ $(9x+2)(9x-2)$	5. $x^2 - 1$ $(x+1)(x-1)$	6. $x^2 - y^2$
7. $49 - a^2$ $(7+a)(7-a)$	8. $36x^2 - 121y^2$	9. $25x^4 - 4$ $(5x^2+2)(5x^2-2)$	10. $196 - 9a^4$
11. $225p^2 - 100$ $(15p+10)(15p-10)$	12. $625 - 121w^2$ $(25+11w)(25-11w)$	13. $25x^6 - 81$	14. $400y^2 - 900a^2$
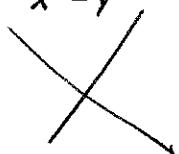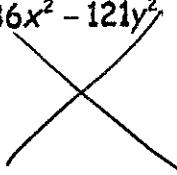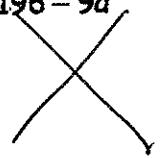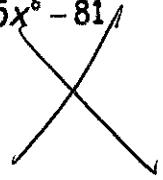
To factor a polynomial <u>completely</u> means to apply EVERY factoring technique that applies in a problem. *Please refer to the Factoring Flowchart given in class.*

Examples

15. $3x - 12$	16. $x^2 - 16$	17. $5x^4 - 405$
$3(x-4)$	$(x+4)(x-4)$ $x = -4, 4$	
18. $x^2 + 16$ $x^2 + 16$ NO solution	19. $3x^2 - 12x + 3$ $3(x^2 - 4x + 1)$ $x = 3.73, .268$	20. $12x^2 - x - 6$ $(4x-3)(3x+2)$ $x = \frac{3}{4}, \frac{-2}{3}$
21. $4x^2 - 20x + 16$ $4(x-4)(x-1)$ $x = 4, 1$	22. $12ab^2 + 10cb^2 - 12ad^2 - 10cd^2$	23. $2x^2 + 13x - 7$ $(x+7)(2x-1)$ $x = -7, \frac{1}{2}$
24. $5m^2 + 20m + 40$ $5(m^2 + 4m + 8)$ NO solution	25. $3x^5y - 243x^3y$	26. $-10x^2 - 5x + 75$ $-5(2x-5)(x+3)$ $x = \frac{5}{2}, -3$

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Simplify Radical Expressions

A radical Expression is in simplest form when the following conditions are met:

1._____

2._____

3._____

Properties of Radical Expressions: Examples:

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	4. $\sqrt{3x} \cdot \sqrt{20x}$
$\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$	5. $\sqrt{\dfrac{x^2}{100}}$

Simplify each of the following.

6. $\sqrt{72}$	7. $\sqrt{3x^2}$	8. $\sqrt{45y^5}$	9. $-3\sqrt{12x^2}$
$6\sqrt{2}$	$x\sqrt{3}$	$3y^2\sqrt{5y}$	$-6x\sqrt{3}$
10. $\sqrt{5} \cdot \sqrt{10}$	11. $-2\sqrt{3x} \cdot 4\sqrt{15xy}$	12. $\sqrt{\dfrac{5}{81}}$	13. $\sqrt{\dfrac{2x^4}{9y^2}}$
$5\sqrt{2}$	$-24x\sqrt{5y}$	$\dfrac{\sqrt{5}}{9}$	$\dfrac{x^2\sqrt{2}}{3y}$

Rationalizing the denominator:
A process used to eliminate a radical from the denominator of a fraction.

14. $\dfrac{5}{\sqrt{7}}$ $\dfrac{5\sqrt{7}}{7}$	15. $\sqrt{\dfrac{5}{7}}$ $\dfrac{\sqrt{35}}{7}$
16. $\sqrt{\dfrac{3x}{50}}$ $\dfrac{\sqrt{6x}}{10}$	17. $\dfrac{9}{\sqrt{2x}}$ $\dfrac{9\sqrt{2x}}{2x}$

Sums and Differences of Radical Expressions: Combine Like Radicals!!!

18. $4\sqrt{10}+\sqrt{10}-9\sqrt{10}$ $-4\sqrt{10}$	19. $5\sqrt{3}+\sqrt{48}$ $9\sqrt{3}$
20. $\sqrt{12}+6\sqrt{3}-\sqrt{20}$ $8\sqrt{3}-2\sqrt{5}$	21. $7\sqrt{28}+\sqrt{63}-2\sqrt{7}$ $15\sqrt{7}$

Name_____

Multiplying Radical Expressions $\quad$ *odds only*

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Simplify.

1) $3\sqrt{12} \cdot \sqrt{6}$ $\quad 18\sqrt{2}$

2) $\sqrt{5} \cdot \sqrt{10}$

3) $\sqrt{6} \cdot \sqrt{6}$ $\quad 6$

4) $\sqrt{5} \cdot -4\sqrt{20}$

5) $-4\sqrt{15} \cdot -\sqrt{3}$ $\quad 12\sqrt{5}$

6) $\sqrt{20x^2} \cdot \sqrt{20x}$

7) $\sqrt{15n^2} \cdot \sqrt{10n^3}$ $\quad 5n^2\sqrt{6n}$

8) $\sqrt{18a^2} \cdot 4\sqrt{3a^2}$

9) $-3\sqrt{7r^3} \cdot 6\sqrt{7r^2}$ $\quad -126r^2\sqrt{r}$

10) $-4\sqrt{28x} \cdot \sqrt{7x^3}$

11) $\sqrt{3}(5 + \sqrt{3})$ $\quad 5\sqrt{3}+3$

12) $2\sqrt{5}(\sqrt{6} + 2)$

13) $-3\sqrt{3}(2 + \sqrt{6})$ $\quad -6\sqrt{3}-9\sqrt{2}$

14) $\sqrt{3}(-5\sqrt{10} + \sqrt{6})$

15) $-2\sqrt{15}(-3\sqrt{3}+3\sqrt{5})$

$$18\sqrt{5}-30\sqrt{3}$$

16) $5\sqrt{42x}(4+4\sqrt{7x})$

17) $\sqrt{14x}(3-\sqrt{2x})$

$$3\sqrt{14x}-2x\sqrt{7}$$

18) $\sqrt{6n}(7n^3+\sqrt{12})$

19) $\sqrt{3v}(\sqrt{6}+\sqrt{10})$

$$3\sqrt{2v}+\sqrt{30}v$$

20) $\sqrt{21r}(5+\sqrt{7})$

21) $(-2\sqrt{3}+2)(\sqrt{3}-5)$

$$-16+12\sqrt{3}$$

22) $(5-4\sqrt{5})(-2+\sqrt{5})$

23) $(-2-3\sqrt{5})(5-\sqrt{5})$

$$5-13\sqrt{5}$$

24) $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$

25) $(5\sqrt{2x}+\sqrt{5})(-4\sqrt{2x}+\sqrt{5x})$

$$-40x+5x\sqrt{10}-4\sqrt{10}x+5\sqrt{x}$$

26) $(-3\sqrt{3k}+4)(\sqrt{3k}-5)$

27) $(5+4\sqrt{3})(3+\sqrt{3})$

$$27+17\sqrt{3}$$

28) $(3\sqrt{2}+\sqrt{5})(\sqrt{2}-3\sqrt{5r})$

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Solving Radical Equations

A radical Equation contains a radical expression with the variable in the radicand. To Solve:

1._____

2._____

Examples of squaring both sides of an equation:

1. $\sqrt{x} = 4$	2. $\sqrt{2x} = \sqrt{10}$

Squaring both sides of an equation sometimes results in **EXTRANEOUS VALUES**. Check your final answers to be sure they both work!! Only list the solutions true solutions.

Examples:

3. $3\sqrt{x} - 6 = 0$	4. $3\sqrt{4x-19} - 11 = 10$
5. $\sqrt{3x-3} = \sqrt{2x+8}$	6. $\sqrt{6-x} = x$

More examples:

7. $\sqrt{x+6} - \sqrt{x-6} = 0$	8. $\sqrt{4x-19} - \sqrt{2x+12} = 0$
9. $\sqrt{20-x} - x = 0$	10. $4\sqrt{x-7} + 12 = 28$