

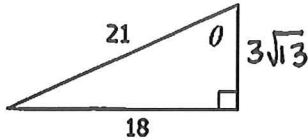
Unit 12 Test Study Guide (Trigonometry - Part 1)

Name: _____

Date: _____ Block: _____

Topic 1: Trigonometric Functions

1. Find the values of the six trigonometric functions for angle θ . Give answers in simplest form.



$$x^2 + 18^2 = 21^2$$

$$x^2 + 324 = 441$$

$$x^2 = 117$$

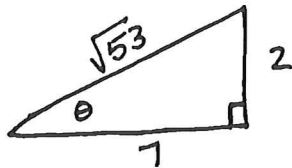
$$x = 3\sqrt{13}$$

$$\sin \theta = \frac{18}{21} = \frac{6}{7} \quad \csc \theta = \frac{21}{18} = \frac{7}{6}$$

$$\cos \theta = \frac{3\sqrt{13}}{21} = \frac{\sqrt{13}}{7} \quad \sec \theta = \frac{21}{3\sqrt{13}} = \frac{7\sqrt{13}}{13}$$

$$\tan \theta = \frac{18}{3\sqrt{13}} = \frac{6\sqrt{13}}{13} \quad \cot \theta = \frac{3\sqrt{13}}{18} = \frac{\sqrt{13}}{6}$$

2. If $\tan \theta = \frac{2}{7}$, find the remaining trigonometric functions.



$$2^2 + 7^2 = x^2$$

$$4 + 49 = x^2$$

$$53 = x^2$$

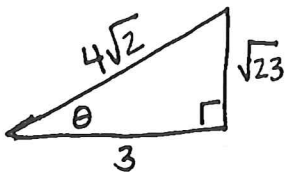
$$\sqrt{53} = x$$

$$\sin \theta = \frac{2}{\sqrt{53}} = \frac{2\sqrt{53}}{53} \quad \csc \theta = \frac{\sqrt{53}}{2}$$

$$\cos \theta = \frac{7}{\sqrt{53}} = \frac{7\sqrt{53}}{53} \quad \sec \theta = \frac{\sqrt{53}}{7}$$

$$\tan \theta = \frac{2}{7} \quad \cot \theta = \frac{7}{2}$$

3. If $\sec \theta = \frac{4\sqrt{2}}{3}$, find the remaining trigonometric functions.



$$3^2 + x^2 = (4\sqrt{2})^2$$

$$9 + x^2 = 32$$

$$x^2 = 23$$

$$x = \sqrt{23}$$

$$\sin \theta = \frac{\sqrt{23}}{4\sqrt{2}} = \frac{\sqrt{46}}{8} \quad \csc \theta = \frac{4\sqrt{2}}{\sqrt{23}} = \frac{4\sqrt{46}}{23}$$

$$\cos \theta = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8} \quad \sec \theta = \frac{4\sqrt{2}}{3}$$

$$\tan \theta = \frac{\sqrt{23}}{3} \quad \cot \theta = \frac{3}{\sqrt{23}} = \frac{3\sqrt{23}}{23}$$

Directions: Find each missing measure to the nearest tenth.

4. $\sin 42 = \frac{x}{14}$
 $x = 9.4$

5. $\sin 18 = \frac{21}{x}$
 $x = \frac{21}{\sin 18}$
 $x = 68$

6. $\cos \theta = \frac{19}{23}$
 $\theta = \cos^{-1}(19/23)$
 $\theta = 34.3^\circ$

7. $\sin \theta = \frac{34}{35}$
 $\theta = \sin^{-1}(34/35)$
 $\theta = 76.3^\circ$

Topic 2: Angles & Angle Measures

Directions: Give each degree measure in radians and radian measure in degrees.

8. 165°

$$165 \cdot \frac{\pi}{180} = \frac{165\pi}{180} = \boxed{\frac{11\pi}{12}}$$

9. -54°

$$-54 \cdot \frac{\pi}{180} = \frac{-54\pi}{180} = \boxed{-\frac{3\pi}{10}}$$

10. 243°

$$243 \cdot \frac{\pi}{180} = \frac{243\pi}{180} = \boxed{\frac{27\pi}{20}}$$

11. $\frac{9\pi}{5}$

$$\frac{9\pi}{5} \cdot \frac{180}{\pi} = \frac{1620}{5} = \boxed{324^\circ}$$

12. $-\frac{13\pi}{20}$

$$-\frac{13\pi}{20} \cdot \frac{180}{\pi} = \frac{-2340}{20} = \boxed{-117^\circ}$$

13. $-\frac{4\pi}{45}$

$$-\frac{4\pi}{45} \cdot \frac{180}{\pi} = \frac{-720}{45} = \boxed{-16^\circ}$$

Directions: Give two coterminal \angle 's (one positive and one negative) and a reference \angle for each.

14. 305°



Coterminal \angle 's: $665^\circ, -55^\circ$

Reference \angle : 55°

15. $\frac{7\pi}{10}$ (126°)

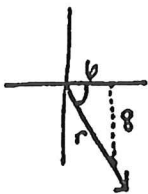


Coterminal \angle 's: $\frac{27\pi}{10}, -\frac{13\pi}{10}$

Reference \angle : $\frac{3\pi}{10}$

Directions: The terminal side of θ in standard position contains each point. Find the exact values of each trigonometric function of θ .

16. (6, -8)



$$6^2 + 8^2 = r^2$$

$$36 + 64 = r^2$$

$$100 = r^2$$

$$10 = r$$

$$x = 6$$

$$y = -8$$

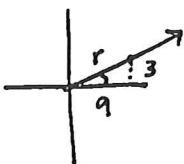
$$r = 10$$

$$\sin \theta = \frac{-8}{10} = -\frac{4}{5} \quad \csc \theta = -\frac{5}{4}$$

$$\cos \theta = \frac{6}{10} = \frac{3}{5} \quad \sec \theta = \frac{5}{3}$$

$$\tan \theta = \frac{-8}{6} = -\frac{4}{3} \quad \cot \theta = -\frac{3}{4}$$

17. (9, 3)



$$9^2 + 3^2 = r^2$$

$$81 + 9 = r^2$$

$$90 = r^2$$

$$3\sqrt{10} = r$$

$$x = 9$$

$$y = 3$$

$$r = 3\sqrt{10}$$

$$\sin \theta = \frac{3}{3\sqrt{10}} = \frac{\sqrt{10}}{10} \quad \csc \theta = \frac{\sqrt{10}}{1}$$

$$\cos \theta = \frac{9}{3\sqrt{10}} = \frac{3\sqrt{10}}{10} \quad \sec \theta = \frac{\sqrt{10}}{3}$$

$$\tan \theta = \frac{3}{9} = \frac{1}{3} \quad \cot \theta = 3$$

Topic 3: The Unit Circle

$$\frac{s^+}{r^+} \mid \frac{A^+}{c^+}$$

18. Which trigonometric functions are negative in quadrant IV?

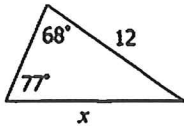
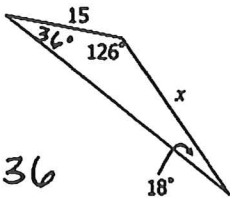
Sine and tangent

19. If $\cos \theta < 0$ and $\csc \theta < 0$ which quadrant(s) could the terminal side of θ lie?

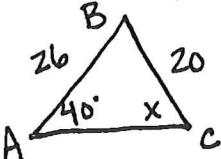
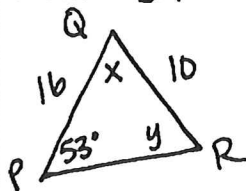
Q3

Directions: Give the exact value of each trigonometric function.		
20. $\sin 45$ $\frac{\sqrt{2}}{2}$	21. $\cot 135^\circ$ ($\tan 135 = -1$) -1	22. $\sec 330^\circ$ ($\cos 330 = \frac{\sqrt{3}}{2}$) $\frac{2\sqrt{3}}{3}$
23. $\tan \frac{4\pi}{3}$ $\sqrt{3}$	24. $\csc \frac{11\pi}{6}$ ($\sin \frac{11\pi}{6} = -1/2$) -2	25. $\cos \frac{2\pi}{3}$ $-\frac{1}{2}$
26. $\tan \frac{7\pi}{6}$ $\frac{\sqrt{3}}{3}$	27. $\sec \frac{5\pi}{4}$ ($\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$) $-\sqrt{2}$	28. $\cot \pi$ ($\tan = 0$) Undefined

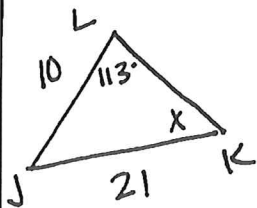
Topic 4: Law of Sines

Directions: Find each missing length to the nearest tenth.	
<p>29.</p> $\frac{\sin 77}{12} = \frac{\sin 68}{x}$  $x \cdot \sin 77 = 12 \cdot \sin 68$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">$x = 11.4$</div>	<p>30.</p> $\frac{\sin 18}{15} = \frac{\sin 36}{x}$  $x \cdot \sin 18 = 15 \cdot \sin 36$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">$x = 28.5$</div>

Directions: Find each missing angle to the nearest tenth. Give all possible solutions.

<p>31. In $\triangle ABC$, $m\angle A = 40^\circ$, $AB = 26$, and $BC = 20$. Find $m\angle C$.</p>  $\frac{\sin 40}{20} = \frac{\sin x}{26}$ $20 \cdot \sin x = 26 \cdot \sin 40$ $\sin x = .8356$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">$x = 56.7^\circ, 123.3^\circ$</div> <div style="float: right; margin-top: 10px;"> $40 + 56.7 < 180 \checkmark$ $40 + 123.3 < 180 \checkmark$ </div>
<p>32. In $\triangle PQR$, $m\angle P = 53^\circ$, $PQ = 16$, and $QR = 10$. Find $m\angle Q$.</p>  $\frac{\sin 53}{10} = \frac{\sin y}{16}$ $16 \cdot \sin 53 = 10 \cdot \sin y$ $\sin y = 1.2778$ $y = \emptyset$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">No Solution</div>

32. In $\triangle JKL$, $m\angle L = 113^\circ$, $JK = 21$, and $JL = 10$. Find $m\angle K$.



$$\frac{\sin 113}{21} = \frac{\sin x}{10}$$

$$10 \cdot \sin 113 = 21 \cdot \sin x$$

$$\sin x = .4383$$

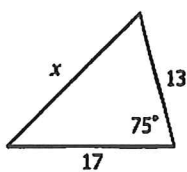
$$x = 26^\circ; 154^\circ$$

26°

Topic 5: Law of Cosines

Directions: Find each missing measure to the nearest tenth.

34.



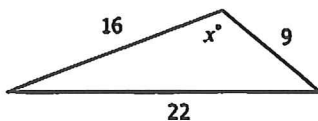
$$x^2 = 17^2 + 13^2 - 2(17)(13) \cdot \cos 75$$

$$x^2 = 458 - 442 \cos 75$$

$$x^2 = 343.6020$$

$$x = 18.5$$

35.



$$22^2 = 16^2 + 9^2 - 2(16)(9) \cdot \cos x$$

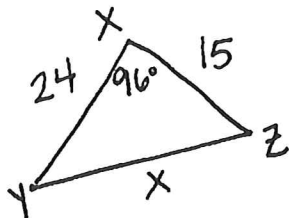
$$484 = 337 - 288 \cdot \cos x$$

$$147 = -288 \cdot \cos x$$

$$-.5104 = \cos x$$

$$x = 120.7^\circ$$

36. In $\triangle XYZ$, $m\angle X = 96^\circ$, $XZ = 15$, and $XY = 24$. Find YZ .



$$x^2 = 24^2 + 15^2 - 2(24)(15) \cdot \cos 96$$

$$x^2 = 801 - 720 \cdot \cos 96$$

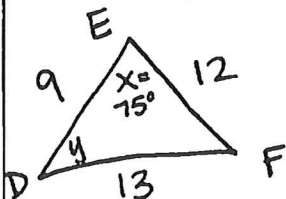
$$x^2 = 876.2605$$

$$x = 29.6$$

Topic 6: Solving Triangles

Directions: Solve each triangle. Round all answers to the nearest tenth.

37. In $\triangle DEF$, $DE = 9$, $EF = 12$, and $DF = 13$.



$$13^2 = 9^2 + 12^2 - 2(9)(12) \cdot \cos x$$

$$169 = 225 - 216 \cdot \cos x$$

$$-56 = -216 \cdot \cos x$$

$$.2593 = \cos x$$

$$x = 75^\circ$$

$$\frac{\sin 75}{13} = \frac{\sin y}{12}$$

$$12 \cdot \sin 75 = 13 \cdot \sin y$$

$$\sin y = .8916$$

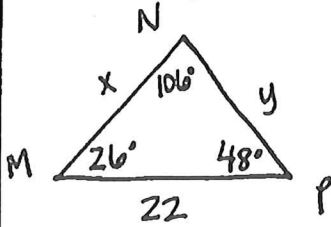
$$y = 43.1^\circ$$

$$m\angle D = 43.1^\circ$$

$$m\angle E = 75^\circ$$

$$m\angle F = 41.9^\circ$$

38. In $\triangle MNP$, $m\angle M = 26^\circ$, $m\angle P = 48^\circ$, and $MP = 22$.



$$\frac{\sin 106}{22} = \frac{\sin 48}{x}$$

$$x \cdot \sin 106 = 22 \cdot \sin 48$$

$$x = 17$$

$$\frac{\sin 106}{22} = \frac{\sin 26}{y}$$

$$y \cdot \sin 106 = 22 \cdot \sin 26$$

$$y = 10$$

$$m\angle N = 106^\circ$$

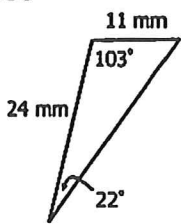
$$MN = 17$$

$$NP = 10$$

Topic 7: Area of Triangles

Directions: Find the area of each triangle to the nearest tenth.

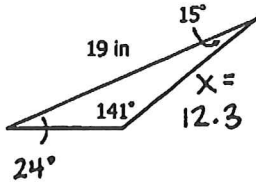
39.



$$A = \frac{1}{2} (24)(11) \cdot \sin 103$$

$$A = 128.6 \text{ mm}^2$$

40.



$$\frac{\sin 141}{19} = \frac{\sin 24}{x}$$

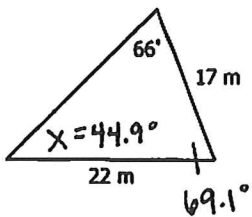
$$x \cdot \sin 141 = 19 \cdot \sin 24$$

$$x = 12.3$$

$$A = \frac{1}{2} (19)(12.3) \cdot \sin 15$$

$$A = 30.2 \text{ in}^2$$

41.



$$\frac{\sin 66}{22} = \frac{\sin x}{17}$$

$$22 \cdot \sin x = 17 \cdot \sin 66$$

$$\sin x = .7059$$

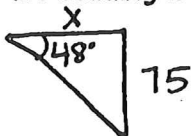
$$x = 44.9^\circ, 135.1$$

$$A = \frac{1}{2} (22)(17) \cdot \sin 69.1$$

$$A = 174.7 \text{ m}^2$$

Topic 8: Applications

42. From the top of a building, the angle of depression to a car parked on the street below is 48° . If the building is 75 feet tall, how far is the car parked from the base of the building?



$$\tan 48 = \frac{75}{x}$$

$$x = \frac{75}{\tan 48}$$

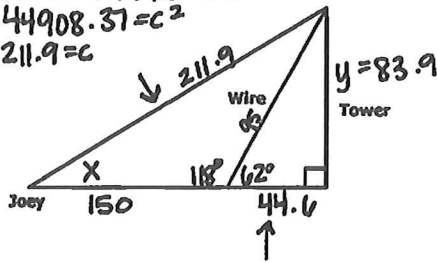
$$x = 67.5 \text{ ft}$$

43. A 95-foot wire attached from the top of a cell phone tower makes a 62° angle with the ground. Joey is standing 150 feet from the end of the wire, looking up at the tower. Find the angle of elevation from the point on the ground where Joey is standing to the top of the tower.

$$83.9^2 + 194.6^2 = c^2$$

$$44908.37 = c^2$$

$$211.9 = c$$



$$a^2 + 83.9^2 = 95^2$$

$$a^2 = 1985.79$$

$$a = 44.6$$

$$\frac{\sin 62}{y} = \frac{\sin 90}{95}$$

$$y \cdot \sin 90 = 95 \cdot \sin 62$$

$$y = 83.9$$

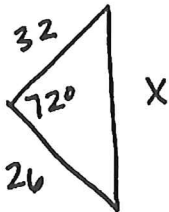
$$\frac{\sin 118}{211.9} = \frac{\sin X}{95}$$

$$211.9 \sin X = 95 \cdot \sin 118$$

$$\sin X = 0.3958$$

$$X = 23.3^\circ$$

44. Two trains leave from the same point and travel along straight tracks that differ in direction by 72° . If their speeds are 64 miles per hour and 52 miles per hour, approximate how far apart they will be after 30 minutes.



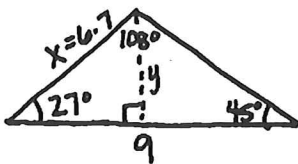
$$X^2 = 32^2 + 26^2 - 2(32)(26) \cdot \cos 72$$

$$X^2 = 1700 - 1664 \cdot \cos 72$$

$$X^2 = 1185.7957$$

$$X = 34.4 \text{ mi}$$

45. The angles of elevation to a balloon in the air from two points on level ground are 27° and 45° . If the points are 9 miles apart and the balloon is between the points, in the same vertical plane, approximate, to the nearest tenth of a mile, the height of the balloon above the ground.



$$\frac{\sin 45}{x} = \frac{\sin 108}{9}$$

$$x \cdot \sin 108 = 9 \cdot \sin 45$$

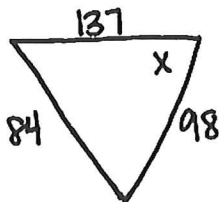
$$x = 6.7$$

$$\frac{\sin 90}{6.7} = \frac{\sin 27}{y}$$

$$y \cdot \sin 90 = 6.7 \cdot \sin 27$$

$$y = 3 \text{ mi}$$

46. A triangular lot has sides of 98 feet, 137 feet, and 84 feet. Find the area of the lot to the nearest tenth.



$$84^2 = 98^2 + 137^2 - 2(98)(137) \cos X$$

$$7056 = 28373 - 26852 \cos X$$

$$-21317 = -26852 \cos X$$

$$.7939 = \cos X$$

$$37.5^\circ = X$$

$$A = \frac{1}{2}(98)(137) \sin 37.5$$

$$A = 4086.6 \text{ ft}^2$$