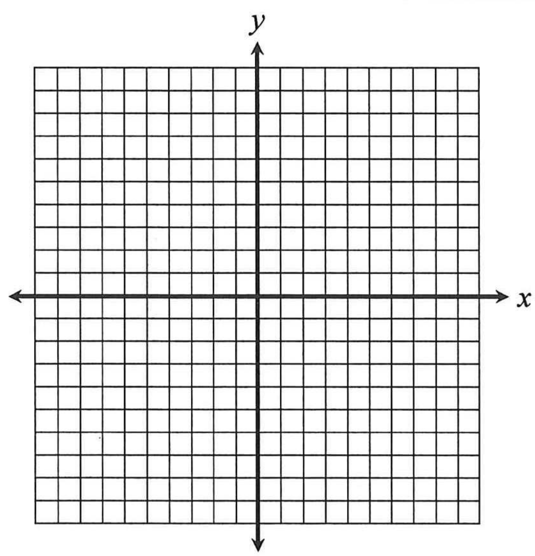
Name:	Date:
Topic:	**Class:**

Main Ideas/Questions	Notes/Examples
EXPONENTIAL *Parent Function*	• If $b > 1$, the function is an ________________ ________________ and is ________________. • If $b < 1$, the function is an ________________ ________________ and is ________________.
ASYMPTOTE	

Directions: Classify as an exponential growth or decay, graph, then identify its key characteristics.

1. $f(x) = 2^x$



Domain: ________________

Range: ________________

End Behavior:

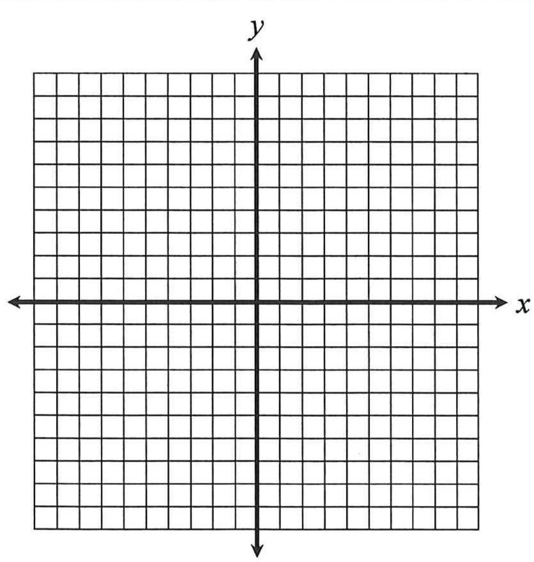
As $x \to \infty$, $f(x) \to$ ______

As $x \to -\infty$, $f(x) \to$ ______

y-intercept: ________________

Asymptote: ________________

2. $f(x) = 3^x$



Domain: ________________

Range: ________________

End Behavior:

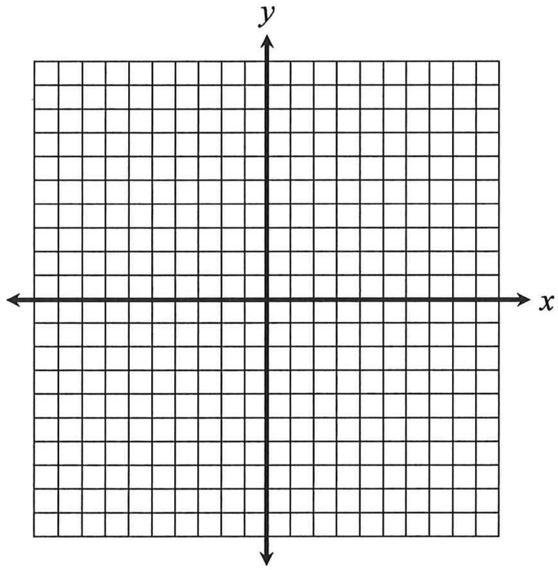
As $x \to \infty$, $f(x) \to$ ______

As $x \to -\infty$, $f(x) \to$ ______

y-intercept: ________________

Asymptote: ________________

3. $f(x)=\left(\dfrac{1}{2}\right)^x$



Domain: ____________________

Range: ____________________

End Behavior:

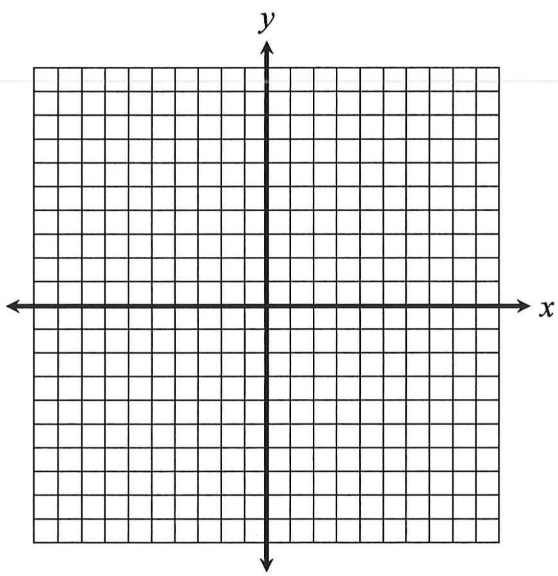
As $x\to\infty$, $f(x)\to$ ______

As $x\to-\infty$, $f(x)\to$ ______

y-intercept: ____________________

Asymptote: ____________________

4. $f(x)=\left(\dfrac{2}{3}\right)^x$



Domain: ____________________

Range: ____________________

End Behavior:

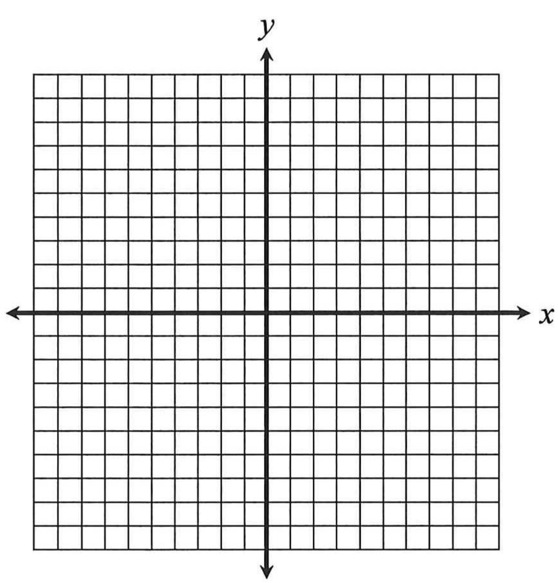
As $x\to\infty$, $f(x)\to$ ______

As $x\to-\infty$, $f(x)\to$ ______

y-intercept: ____________________

Asymptote: ____________________

5. $f(x)=\left(\dfrac{5}{2}\right)^x$



Domain: ____________________

Range: ____________________

End Behavior:

As $x\to\infty$, $f(x)\to$ ______

As $x\to-\infty$, $f(x)\to$ ______

y-intercept: ____________________

Asymptote: ____________________

Name: | **Date:**

Topic: | **Class:**

Main Ideas/Questions	Notes/Examples
TRANFORMATIONS *of Exponential Functions* $f(x) = a \cdot b^{x-h} + k$	• h is the ________________ shift. (+ shifts ______, – shifts ______) • k is the ________________ shift. (+ shifts ______, – shifts ______) • If a is negative, the function is ______________ across the ___-______ • $\lvert a \rvert > 1$ represents a vertical ________________. • $0 < \lvert a \rvert < 1$ represents a vertical ________________.

Directions: (a) Identify the parent function, and **(b)** describe the transformations.

1. $f(x) = 3^x + 5$

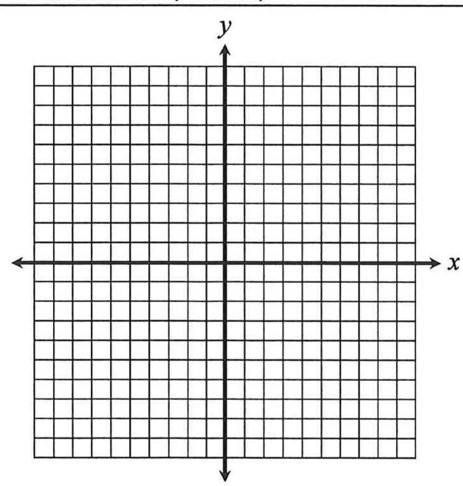
2. $f(x) = 2 \cdot \left(\frac{1}{4}\right)^{x-1}$

3. $f(x) = -\left(\frac{4}{3}\right)^{x+2} + 7$

4. $f(x) = \frac{1}{2} \cdot 5^{x-4} - 2$

Directions: Graph each function and identify its key characteristics.

5. $f(x) = 2^{x+5}$



Domain: ________________

Range: ________________

End Behavior:

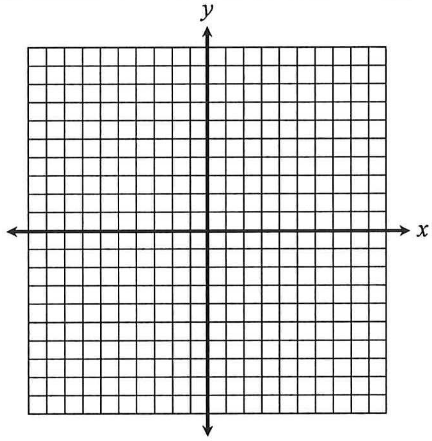
As $x \to \infty$, $f(x) \to$ ______

As $x \to -\infty$, $f(x) \to$ ______

y-intercept: ________________

Asymptote: ________________

6. $f(x)=\left(\dfrac{1}{3}\right)^{x}-2$



Domain: ____________________

Range: ____________________

End Behavior:

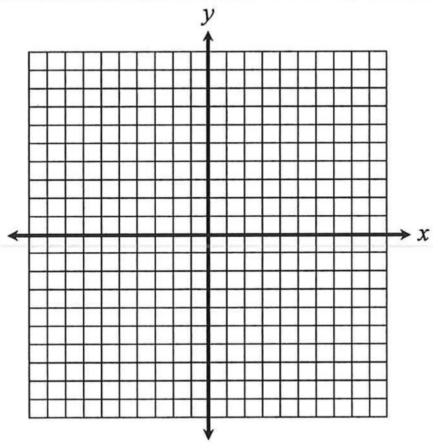
As $x\to\infty$, $f(x)\to$ ______

As $x\to-\infty$, $f(x)\to$ ______

y-intercept: ____________________

Asymptote: ____________________

7. $f(x)=\dfrac{1}{2}\cdot3^{x}+1$



Domain: ____________________

Range: ____________________

End Behavior:

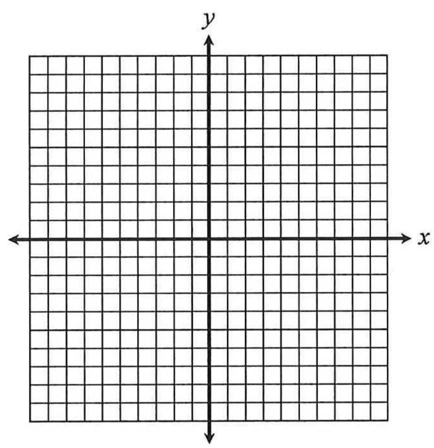
As $x\to\infty$, $f(x)\to$ ______

As $x\to-\infty$, $f(x)\to$ ______

y-intercept: ____________________

Asymptote: ____________________

8. $f(x)=\left(\dfrac{3}{2}\right)^{x-4}-5$



Domain: ____________________

Range: ____________________

End Behavior:

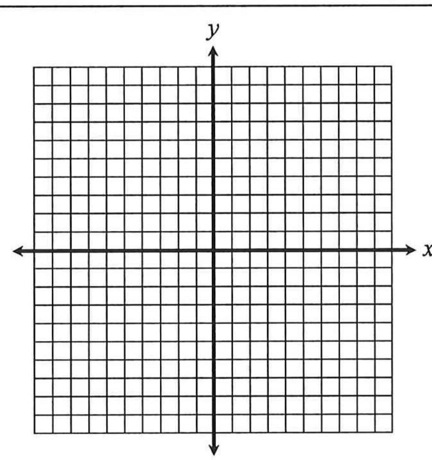
As $x\to\infty$, $f(x)\to$ ______

As $x\to-\infty$, $f(x)\to$ ______

y-intercept: ____________________

Asymptote: ____________________

9. $f(x)=-2\cdot4^{x-2}$



Domain: ____________________

Range: ____________________

End Behavior:

As $x\to\infty$, $f(x)\to$ ______

As $x\to-\infty$, $f(x)\to$ ______

y-intercept: ____________________

Asymptote: ____________________

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
SOLVING EXPONENTIAL EQUATIONS	1	Use the properties of exponents to **SIMPLIFY** each side of the equation.
	2	Rewrite the equation so both sides have the **SAME BASE.**
	3	Drop the bases and **SET THE EXPONENTS EQUAL TO EACH OTHER.**

Type 1 – Equations with a Common Base

1. $2^{x+1} = 2^9$

2. $5^{4n+5} = 5^{n-7}$

3. $3^k \cdot 3^{k+2} = 3^{5k-1}$

4. $10^{-4} \cdot 10^9 = 10^{v+4} \cdot 10^{2v-11}$

Type 2 – Equations without a Common Base

5. $6^{2x-10} = 36$

6. $2^{p-7} = 8$

7. $7^{4x+11} = \frac{1}{7}$

8. $32 = 2^{2m-9}$

9. $27^{2x+6} = 3^{2x}$

10. $4^{y+2} = 16^{y-3}$

11. $125^{y} = 25$	**12.** $16^{3x} = 8^{x+2}$
13. $4^{3x} = 8^{x-1}$	**14.** $81^{2x+5} = \left(\frac{1}{3}\right)^{2x}$
15. $8^{2a-1} = 32^{2a+1}$	**16.** $27^{2x} = 243^{x-2}$
17. $64 = 4 \cdot 4^{4x}$	**18.** $9^{2x+4} \cdot 9^{2x} = \frac{1}{81}$
19. $\frac{1}{7} = 49^{x-5} \cdot 7^{x-9}$	**20.** $4^{2x} \cdot \frac{1}{16} = 4^{6x+18}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples	
What is a **LOGARITHM?**	**A logarithm (log) is another way of writing exponents.** **Logarithmic Form** $\log_b a = x$ ➡ **Exponential Form** Read as "**log base *b* of *a* equals *x*.**"	
Converting **LOG ➡ EXP**	**Directions:** Write each equation in **exponential form**.	
	1. $\log_3 9 = 2$	**2.** $\log_6 216 = 3$
	3. $\log_7 1 = 0$	**4.** $\log_2 16 = 4$
	5. $\log_4 \frac{1}{16} = -2$	**6.** $\log_9 27 = \frac{3}{2}$
Converting **EXP ➡ LOG**	**Directions:** Write each equation in **logarithmic form**.	
	7. $14^2 = 196$	**8.** $3^4 = 81$
	9. $12^1 = 12$	**10.** $36^{\frac{1}{2}} = 6$
	11. $2^{-3} = \frac{1}{8}$	**12.** $8^{\frac{4}{3}} = 16$

COMMON LOGARITHM

A logarithm with base 10 is called a **common logarithm** and can be written without the base.

$\log_{10} x \longrightarrow$

EVALUATING LOGARITHMS

Directions: Use your knowledge of exponents to evaluate the following logarithms.

13. $\log_7 49$	**14.** $\log_3 27$
15. $\log 100$	**16.** $\log_{12} 1$
17. $\log_2 64$	**18.** $\log_3 243$
19. $\log_9 \frac{1}{81}$	**20.** $\log_{64} 4$

CHANGE OF BASE FORMULA

Some logarithms are not as easy to evaluate as those above, and will require the **change of base formula**.

$\log_b a =$

Choose BASE 10 because there is a calculator button for it!

Directions: Evaluate each log using the change of base formula.

21. $\log_{16} 64$	**22.** $\log_8 32$
23. $\log_2 54$	**24.** $\log_{10} 294$
25. $\log_4 136$	**26.** $\log_6 \frac{1}{36}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples		
Product Property	**Condense into a single logarithm. Simplify if possible.**		
$\log_b(m \cdot n) =$	**1.** $\log_2 7 + \log_2 4$	**2.** $\log 25 + \log 4$	**3.** $\log_4 2x + \log_4 4x^2$
	Expand using the product property.		
	4. $\log 6$	**5.** $\log_7 45$	**6.** $\log_2 (5x)$
Quotient Property	**Condense into a single logarithm. Simplify if possible.**		
$\log_b\left(\frac{m}{n}\right) =$	**7.** $\log_3 24 - \log_3 8$	**8.** $\log_2 15 - \log_2 15$	**9.** $\log_4 x^9 - \log_4 x^2$
	Expand using the quotient property.		
	10. $\log_8 4$	**11.** $\log_5 \frac{1}{3}$	**12.** $\log\left(\frac{m}{7}\right)$
Power Property	**Condense into a single logarithm. Simplify if possible.**		
$\log_b m^n =$	**13.** $5 \cdot \log_4 2$	**14.** $7 \cdot \log_2 x$	**15.** $\frac{1}{3} \cdot \log 8$
	Expand using the power property. Simplify if possible.		
	16. $\log_2 8^7$	**17.** $3 \cdot \log 4^{x-1}$	**18.** $\log_7 \sqrt{w}$

Putting it All Together!

CONDENSING LOGS

Directions: Rewrite as a single logarithm. Simplify if possible.

19. $2 \cdot \log 6 - \log 9$

20. $4 \cdot \log_4 a + 2 \cdot \log_4 b$

21. $7 \cdot \log_4 u - 3 \cdot \log_4 v^2$

22. $\log_2 15 + \log_2 4 - \log_2 6$

23. $\log_3 4 + \log_3 y + \frac{1}{2} \cdot \log_3 49$

24. $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

25. $3 \cdot \log_2 4 - \log_2 32$

26. $2 \cdot \log 6 - \frac{1}{4} \cdot \log 16 + \log 3$

EXPANDING LOGS

Directions: Expand each logarithm.

27. $\log_6 \left(xyz^4\right)$

28. $\log_4 \left(\frac{a^9}{b}\right)$

29. $\log_7 \left(q^4 r^2\right)^2$

30. $\log_2 \left(\frac{y}{z^5}\right)^2$

31. $\log \sqrt{7x^3}$

32. $\log_3 \sqrt[4]{m^5 n^2}$

PROPERTIES OF LOGARITHMS

GRAPHIC ORGANIZER

Name	Rule(s)	Example 1	Example 2
BASIC LOGARITHMS	$\log_b b =$; $\log_b 1 =$	**Simplify:** $\log_{14} 14 =$	**Simplify:** $\log_3 1 =$
PRODUCT RULE	$\log_b (m \cdot n) =$	**Condense:** $\log_5 6 + \log_5 7 =$	**Expand:** $\log_2 63 =$
QUOTIENT RULE	$\log_b \left(\frac{m}{n}\right) =$	**Condense:** $\log_4 84 - \log_4 12 =$	**Expand:** $\log 9 =$
POWER RULE	$\log_b m^n =$	**Condense:** $2 \cdot \log_3 8 =$	**Expand:** $\log_2 6^{x-1} =$
CHANGE OF BASE FORMULA	$\log_b a =$	**Using a common base, evaluate the expression below.** $\log_7 32 =$	

REMEMBER: BASE 10 LOGS ARE COMMON LOGS AND WRITTEN WITHOUT A BASE! **($\log x$)**

Name:

Date:

Topic:

Class:

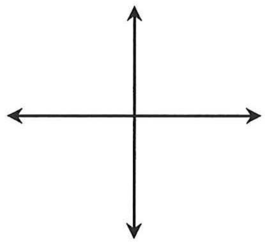
Main Ideas/Questions	Notes/Examples

LOGARITHMIC

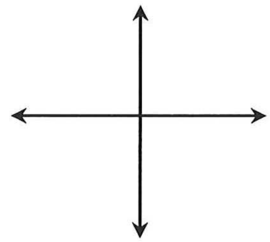
Parent Function

A logarithmic function is the **inverse** of an exponential function.
Using your graphing calculator, sketch the following graphs:

$f(x) = \log x$



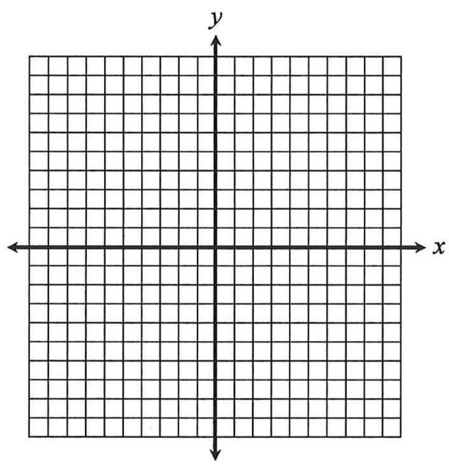
$f(x) = 10^x$



Because you can only graph base 10 logs on your calculator, you will need to use the inverse exponential function, then invert the values from the table to graph the logarithmic function.

Directions: Graph each function and identify its key characteristics.

1. $f(x) = \log_2 x$



Domain: ____________________

Range: ____________________

End Behavior:

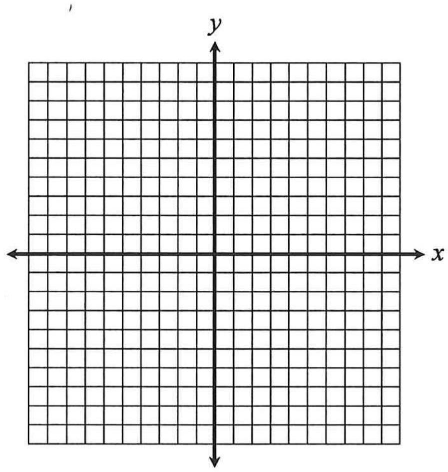
As $x \to$ _______, $f(x) \to \infty$

As $x \to$ _______, $f(x) \to -\infty$

x-intercept: ____________________

Asymptote: ____________________

2. $f(x) = \log_{\frac{1}{3}} x$



Domain: ____________________

Range: ____________________

End Behavior:

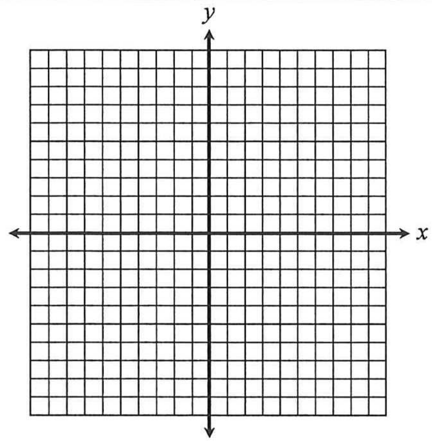
As $x \to$ _______, $f(x) \to \infty$

As $x \to$ _______, $f(x) \to -\infty$

x-intercept: ____________________

Asymptote: ____________________

3. $f(x) = \log_4(x-1)$



Domain: ________________

Range: ________________

End Behavior:

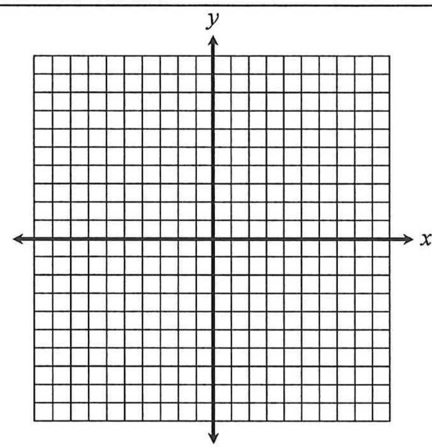
As $x \to$ ______, $f(x) \to \infty$

As $x \to$ ______, $f(x) \to -\infty$

x-intercept: ________________

Asymptote: ________________

4. $f(x) = \log_3 x - 2$



Domain: ________________

Range: ________________

End Behavior:

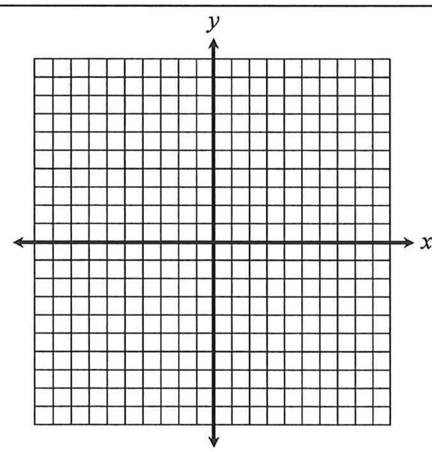
As $x \to$ ______, $f(x) \to \infty$

As $x \to$ ______, $f(x) \to -\infty$

x-intercept: ________________

Asymptote: ________________

5. $f(x) = \log_{\frac{1}{2}}(x+9) + 4$



Domain: ________________

Range: ________________

End Behavior:

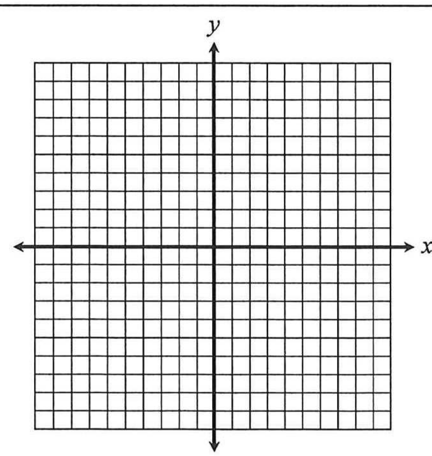
As $x \to$ ______, $f(x) \to \infty$

As $x \to$ ______, $f(x) \to -\infty$

x-intercept: ________________

Asymptote: ________________

6. $f(x) = \log_5(x-2) + 1$



Domain: ________________

Range: ________________

End Behavior:

As $x \to$ ______, $f(x) \to \infty$

As $x \to$ ______, $f(x) \to -\infty$

x-intercept: ________________

Asymptote: ________________

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
Logarithmic Equations **TYPE I: LOG = LOG**	① **CONDENSE** each logarithm. ② **Use the One-to-One Property:** If $\log_b m = \log_b n$, then ③ **SOLVE** and **CHECK FOR EXTRANEOUS SOLUTIONS.**

1. $\log_5(5x+9) = \log_5(6x)$	**2.** $\log_2(1-4n) = \log_2(2n+43)$
3. $\log_9(6-3w) = \log_9(-2w)$	**4.** $\log(y+5) + \log 4 = \log 72$
5. $3 \cdot \log_7 4 = \log_7(4a-8)$	**6.** $\log_4 68 - \log_4 4 = \log_4(3n+11)$
7. $\frac{1}{2} \cdot \log_6 25 = \log_6(23-4w)$	**8.** $\log_3(2p-5) = 2 \cdot \log_3 6 - \log_3 4$

	9. $\log_4(m^2) = \log_4(18-7m)$	**10.** $\log 2 + \log(k^2) = \log(k^2+16)$

TYPE 2: LOG = NUMBER

1	**CONDENSE** and **ISOLATE** the logarithm.
2	Write the equation in **EXPONENTIAL FORM.**
3	**SOLVE** and **CHECK FOR EXTRANEOUS SOLUTIONS.**

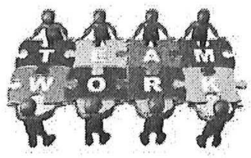
11. $\log_2(x-4) = 6$	**12.** $\log_3(4x+8) - 7 = -3$
13. $\log(2x) + \log(x-5) = 2$	**14.** $2 \cdot \log x - \log 4 = 2$
15. $\log_6(x+9) + \log_6 x = 2$	**16.** $\log(x-3) + \log x = 1$

Name:	**Date:**
Topic:	**Class:**

Main Ideas/Questions	Notes/Examples		
WARM-UP Using a common base to solve an exponential equation.	**Directions:** Solve the equations below using a common base. **1.** $5^{n+10} = 25$	**2.** $9^{a+2} = 27^{4a-2}$	
What if a common base is NOT possible?	①	**ISOLATE** the exponential expression.	
	②	**TAKE THE LOG** of **both sides.**	
	③	You may need to **EXPAND** the log. (Use the Power Rule)	
	④	**SOLVE** and **CHECK FOR EXTRANEOUS SOLUTIONS.**	
	*Rounded answers may not produce the exact same answer, but will be very close.		
Examples	**3.** $2^{x} = 61$	**4.** $8^{m-7} = 92$	
	5. $4 \cdot 7^{n} = 148$	**6.** $4^{3w} - 5 = 3$	

	7. $7-4^{x+1}=18$	**8.** $10\cdot 5^{3k-3}=40$
	9. $4\cdot 3^{n}+15=359$	**10.** $-2\cdot 5^{p}+7=-63$
	11. $5\cdot 9^{v-1}+1=181$	**12.** $8\cdot 11^{7k}-3=213$
	13. $6\cdot 16^{7y+2}-2=82$	**14.** $3\cdot 8^{3-7n}+10=94$

Group Members:	Per: ________



LOGARITHMIC & EXPONENTIAL EQUATIONS REVIEW

Directions: Work **together** to sole each equation. Do not divide up the work! Each person should be participating. At the end of class, one person's paper will be chosen at random and graded for the group.

LOGARITHMIC EQUATIONS

1. $\log_7(9x-4)=\log_7(x+20)$	**2.** $\log_5(m^2-12)=\log_5 m$
3. $\log_3 4+\log_3(a+5)=\log_3 56$	**4.** $\log(2y-10)=7\cdot\log 2-\log 8$
5. $\log_4(5m+9)=3$	**6.** $\log_{36}(20-4p)=\frac{1}{2}$
7. $\log_6(7k-1)=3$	**8.** $\log(n+8)+\log 4=2$

EXPONENTIAL EQUATIONS

9. $25^{v-2} = 625$	**10.** $\frac{1}{16} = 8^{4x-2}$
11. $8^{k} = 78$	**12.** $9^{m-6} = 78$
13. $15^{3a} + 7 = 67$	**14.** $14^{3-8x} + 9 = 77$
15. $8 \cdot 3^{n-1} - 21 = 51$	**16.** $2 \cdot 18^{10r-3} - 1 = 73$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
What is "e"?	• e is an __________ __________ with an approximate value of __________. • e often occurs as the **base of exponential and logarithmic functions** that describe **real-world scenarios**.
Base "e" Exponential Functions	• Exponential functions with base e are called __________ __________ exponential functions. • Example: __________
Base "e" Logarithmic Functions	• Logarithmic functions with base e are called __________ __________ • Example: __________. This is often abbreivated as __________.

Converting Between Forms

Write each equation in logarithmic form.

1. $e^x = 24$	**2.** $e^9 = x$	**3.** $e^{x+5} = 72$

Write each equation in exponential form.

4. $\ln x = 58$	**5.** $\ln 6 = x$	**6.** $\ln(x-9) = 32$

Simplifying with Properties

Condense each expression into a single logarithm.

7. $\ln 3 + \ln 16$	**8.** $\ln 63 - 2 \cdot \ln 3$	**9.** $\frac{1}{3} \cdot \ln 64 + 2 \cdot \ln x$

Expand each logarithm.

10. $\ln 5x$	**11.** $\ln\left(\frac{a^3}{b}\right)^2$	**12.** $\ln \sqrt[3]{m^2 n}$

Solving Equations

Solve each equation below. Check for extraneous solutions.

13. $\ln(4x-27)=\ln(15-2x)$	**14.** $2\cdot\ln k=\ln(2k+15)$
15. $\ln 72-\ln 4=\ln 6+\ln(a-2)$	**16.** $2\cdot\ln(m+4)=\ln 4$
17. $\ln 8x=2$	**18.** $\ln x-\ln 9=7$
19. $e^{x}=57$	**20.** $e^{y+3}-6=24$
21. $5e^{4n}=95$	**22.** $2e^{c-9}+3=87$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
Exponential Growth	Occurs when a **quantity exponentially increases** over time. **Formula:** $a =$ ______ $r =$ ______ $t =$ ______
Examples	**1.** The original value of an investment is $1,800. If the value has increased by 7% each year, write an exponential function to model the situation. Then, find the value of the investment after 15 years.
	2. In 2002, there were 972 students enrolled at Oakview High School. Since then, the number of students has increased by 1.5% each year. Write an exponential function to model the situation, then find the number of students enrolled in 2014.
Exponential Decay	Occurs when a **quantity exponentially decreases** over time. **Formula:** $a =$ ______ $r =$ ______ $t =$ ______
Examples	**3.** An investment of $12,000 is losing value at a rate of 4% each year. Write an exponential function to model the situation, then find the value of the investment after 9 years.
	4. Mark bought a brand new car for $35,000 in 2008. If the car depreciates in value approximately 8% each year, write an exponential function to model the situation. Then, find the value of the car in 2015.

Compound Interest	Occurs when interest is calculated on both the **principal amount AND the accrued interest** thus far. **Formula:** P = ________ r = ________ n = ________ t = ________
Examples	**5.** Laura deposited $12,000 into an account that earns 8% interest. How much money will she have in 5 years if the interest is compounded quarterly?
	6. Jack took out a 6-year loan for $25,000 to purchase a boat at a 4.5% interest rate. If the interest is compounded monthly, what wil he have paid total over the course of the loan?
	7. An investment account pays 3.9% interest compounded semi-annually. If $4,000 is invested in this account, what will be the balance after 12 years?
	8. A savings account offers 0.8% interest compounded bimonthly. If Bob deposited $300 into this account, how much interest will he earn after 10 years?
	9. Suppose you invest $750 into an account that pays 3% interest compounded weekly. How much interest will you have earned after 20 years?

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

Year	Population
1900	3.1
1920	4.6
1940	6.4
1960	9.5
1980	14.2

1. The table to the left shows the year and population (in millions) of Texas. Using an **exponential model**, write an equation for the curve of best fit, then estimate the population of Texas in 2020.

Year	Closing Price ($)
1992	633
1994	793
1996	1,100
1998	1,771
2000	4,696

2. The table to the left sows the closing prices for the NASDAQ Stock Index at the end of February during certain years. Using an **exponential model,** write an equation for the curve of best fit, then estimate the year the closing price will reach $10,000

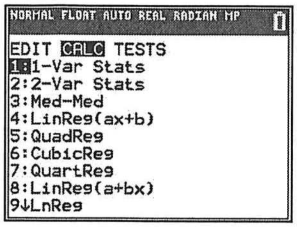
Year	Population
1970	30.0
1980	35.3
1985	37.9
1990	37.2
2006	41.5

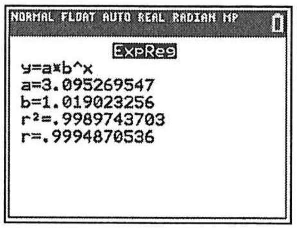
3. The table to the left gives the percent of all workers who are female during selected years from 1970 to 2006. Using a **logarithmic model,** write an equation for the curve of best fit, then estimate the percent of female workers in in 2016.

Age	Height
1	4
2	6.3
3	7.9
4	9.2
5	10.3

4. The table to the left shows the height (in feet) of a pine tree in the park. When it was originally planted, it was one year old and had a height of four feet. Using a **logarithmic model,** write an equation for the curve of best fit, determine the age of the tree at 20 feet.

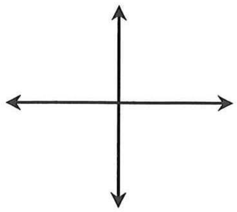
Choosing the Best Model



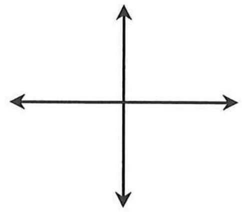


Sketch each parent function.

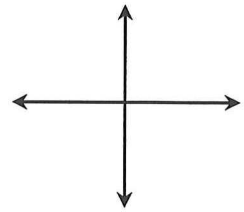
Linear (4:LinReg)



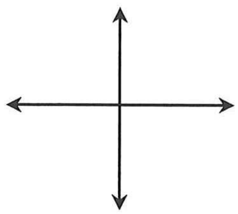
Quadratic (5:QuadReg)



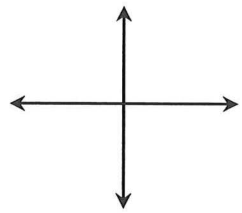
Cubic (6:CubicReg)



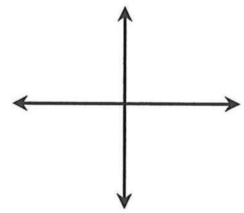
Quartic (7:QuartReg)



Exponential (0:ExpReg)



Logarithmic (9:LnReg)



To **determine the best model**, use the **correlation coeeficient**, r. The closer r is to -1 or 1, the better the fit. Make sure your diagnostics are turned ON to see this value!

5. The table below shows the operating costs (in thousands of dollars) of a small business from 2000 to 2005. Which model would fit this data best: quadratic, quartic, or logarithmic? Using the model, write a best-fit equation, then estimate the operation costs in 2008.

Year	Costs
2000	2.3
2001	2.6
2002	3.1
2003	3.4
2004	4
2005	5.6

6. The table below shows the balance of Marissa's savings account at the end of the year. Which model would fit this data best: linear, cubic, or exponential? Using the model, write a best-fit equation, then estimate the year the balance will reach $1,500.

Year	Balance ($)
2009	500
2010	540
2011	583.20
2012	629.86
2013	680.24
2014	734.66