Unit 7 Test Study Guide
(Exponential & Logarithmic Functions)

Name: _____

Date: _____ Block: _____

Topic 1: Graphing Exponential & Logarithmic Functions

Describe as an exponential growth or decay.

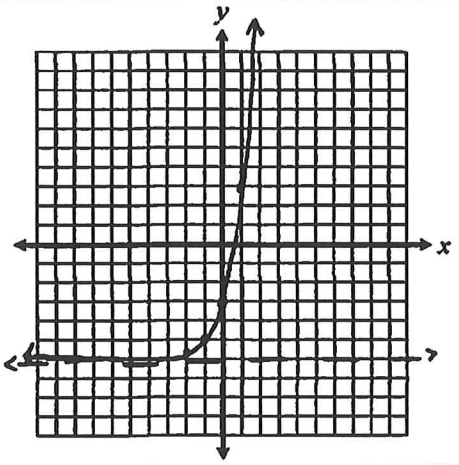
1. $f(x)=5\left(\frac{2}{3}\right)^x$ decay
   ↳ <1

2. $f(x)=\frac{1}{3}\left(\frac{6}{5}\right)^x$ growth
   ↳ >1

Graph each function and identify its key characteristics.

3. $f(x)=3^{x+1}-6$

X	Y
-2	$-5.\overline{6}$
-1	-5
0	-3
1	3
2	21



Domain: $\mathbb{R}$

Range: $y > -6$

End Behavior:

As $x \to \infty$, $f(x) \to$ ∞

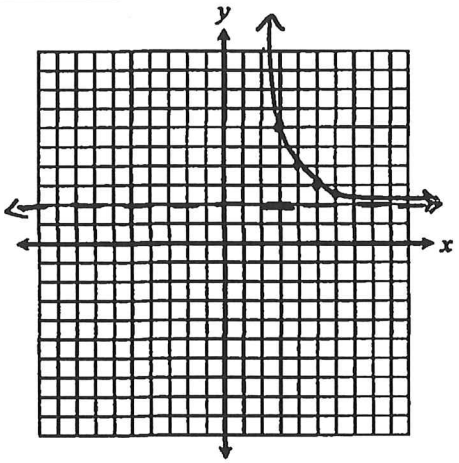
As $x \to -\infty$, $f(x) \to$ -6

y-intercept: $(0,3)$

Asymptote: $y = -6$

4. $f(x)=\left(\frac{1}{2}\right)^{x-5}+2$

X	Y
2	10
3	6
4	4
5	3
6	2.5



Domain: $\mathbb{R}$

Range: $y > 2$

End Behavior:

As $x \to \infty$, $f(x) \to$ 2

As $x \to -\infty$, $f(x) \to$ ∞

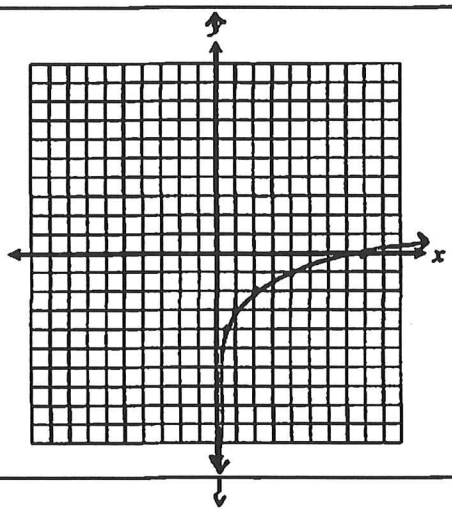
y-intercept: none

Asymptote: $y = 2$

5. $f(x)=\log_2 x-3$

X	Y
0.25	-2
½	-1
1	0
2	1
4	2
8	3

X	Y
0.25	-5
0.5	-4
1	-3
2	-2
4	-1
8	0



Domain: $x > 0$

Range: $\mathbb{R}$

End Behavior:

As $x \to$ ∞, $f(x) \to \infty$

As $x \to$ 0, $f(x) \to -\infty$

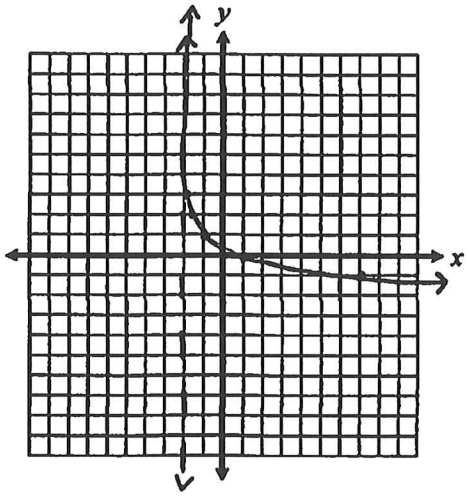
x-intercept: $(8,0)$

Asymptote: $x = 0$

6. $f(x)=\log_{\frac{1}{3}}(x+2)+1$

X	y
9	-2
3	-1
1	0
$0.\bar{3}$	1
$0.\bar{1}$	2

X	y
-7	-1
-1	0
-1	1
$-1.\bar{6}$	2
$-1.\bar{9}$	3



Domain: $X > -2$

Range: $\mathbb{R}$

End Behavior:

As $x \to$ -2, $f(x) \to \infty$

As $x \to$ ∞, $f(x) \to -\infty$

x-intercept: $(1, 0)$

Asymptote: $X = -2$

Topic 2: Exponential vs. Logarithmic Form

Write in logarithmic form.			
7. $8^2 = 64$	**8.** $2^{x-4} = 32$	**9.** $10^{2x} = 54$	**10.** $e^6 = x-2$
$\log_8 64 = 2$	$\log_2 32 = X-4$	$\log_{10} 54 = 2x$	$\ln(x-2) = 6$
Write in exponential form.			
11. $\log_3 27 = 3$	**12.** $\log_x 7 = \frac{1}{2}$	**13.** $\log_4 90 = x$	**14.** $\ln x = 38$
$3^3 = 27$	$x^{1/2} = 7$	$4^x = 90$	$e^{38} = X$

Topic 3: Evaluating Logarithms

Evaluate each logarithm. Use the Change of Base Formula when necessary: $\log_b a = \frac{\log a}{\log b}$

15. $\log_9 81$	**16.** $\log_{81} 3$	**17.** $\log_5 \frac{1}{25}$	**18.** $\log_6 1$
$9^X = 81$	$81^X = 3$	$5^X = 1/25$	$6^X = 1$
$X = 2$	$X = 1/4$	$X = -2$	$X = 0$
19. $\log 63$	**20.** $\log_7 95$	**21.** $\log_2 78$	**22.** $\ln 42$
1.7993	2.3402	6.2854	3.7377

Topic 4: Properties of Logarithms

Product Rule	**Quotient Rule**	**Power Rule**
$\log_b(m \cdot n) =$	$\log_b\left(\frac{m}{n}\right) =$	$\log_b m^n =$
$\log_b m + \log_b n$	$\log_b m - \log_b n$	$n \cdot \log_b m$

Condense each expression into a single logarithm.

23. $3 \cdot \log 2 + \log(x-4)$

$\log 2^3 \cdot (x-4)$

$\boxed{\log(8x-32)}$

24. $\frac{1}{2} \cdot \log_5 324 - \log_5 2$

$\log_5 \frac{324^{1/2}}{2}$

$\boxed{\log_5 9}$

25. $3 \cdot \ln 6 - \frac{3}{2} \cdot \ln 4$

$\ln \frac{6^3}{4^{3/2}}$

$\boxed{\ln 27}$

Expand each expression.

26. $\log_3(x^2y^5)^3$

$\log_3 x^6 y^{15}$

$\boxed{6 \cdot \log_3 x + 15 \cdot \log_3 y}$

27. $\ln\left(\frac{2}{a^3}\right)^4$

$\ln \frac{16}{a^{12}}$

$\boxed{\ln 16 - 12 \cdot \ln a}$

28. $\log_4 \sqrt{p^3 q^{10}}$

$\log_4 p^{3/2} q^5$

$\boxed{\frac{3}{2} \cdot \log_4 p + 5 \cdot \log_4 q}$

Topic 5: Solving Logarithmic Equations

29. $\log_4(5x+7) = \log_4(2x+31)$

$5x+7 = 2x+31$

$3x = 24$

$\boxed{x = 8}$

30. $\ln(p^2 - p) = \ln(6p+18)$

$p^2 - p = 6p + 18$

$p^2 - 7p - 18 = 0$

$(p-9)(p+2) = 0$

$\boxed{p=9}$ | $\boxed{p=-2}$

31. $\frac{1}{2} \cdot \log_8 36 + \log_8(3k+7) = \log_8 132$

$36^{1/2}(3k+7) = 132$

$18k + 42 = 132$

$18k = 90$

$\boxed{k=5}$

32. $2 \cdot \log(y+5) = \log 20 - \log 5$

$(y+5)^2 = 4$

$y^2 + 10y + 25 = 4$

$y^2 + 10y + 21 = 0$

$(y+7)(y+3) = 0$

~~$y=-7$~~ | $\boxed{y=-3}$

33. $\log_2(9m+2)=7$

$2^7 = 9m+2$

$128 = 9m+2$

$126 = 9m$

$\boxed{14 = m}$

34. $5\cdot\ln(2a-1)=15$

$\ln(2a-1)=3$

$e^3 = 2a-1$

$20.09 = 2a-1$

$21.09 = 2a$

$\boxed{10.548 = a}$

Topic 6: Solving Exponential Equations

35. $64^{x+7} = 4^{5x-3}$

$(4^3)^{x+7} = 4^{5x-3}$

$3x+21 = 5x-3$

$24 = 2x$

$\boxed{12 = x}$

36. $9^{w-8} = \left(\frac{1}{27}\right)^{2w}$

$(3^2)^{w-8} = (3^{-3})^{2w}$

$2w-16 = -6w$

$-16 = -8w$

$\boxed{2 = w}$

37. $8^{n-5} = 48$

$\log_8 48 = n-5$

$\frac{\log 48}{\log 8} = n-5$

$1.8617 = n-5$

$\boxed{6.8617 = n}$

38. $2\cdot 3^{4y} - 11 = 61$

$2\cdot 3^{4y} = 72$

$3^{4y} = 36$

$\log_3 36 = 4y$

$\frac{\log 36}{\log 3} = 4y$

$3.2619 = 4y$

$\boxed{y = 0.8155}$

39. $e^{a+1} = 65$

$a+1 = \ln 65$

$a+1 = 4.1744$

$\boxed{a = 3.1744}$

40. $-3\cdot e^{2m-5} - 7 = -34$

$-3\cdot e^{2m-5} = -27$

$e^{2m-5} = 9$

$2m-5 = \ln 9$

$2m-5 = 2.1972$

$2m = 7.1972$

$\boxed{m = 3.5986}$

Topic 7: Applications

Exponential Growth	Exponential Decay	Compound Interest
$A = a(1+r)^t$	$A = a(1-r)^t$	$A = P\left(1+\frac{r}{n}\right)^{nt}$
a = initial amount r = growth rate t = time	a = initial amount r = decay rate t = time (in years)	P = initial amount r = rate n = # of times compounded/year t = time (in years)

41. Sophie invested $5,000 into an account that will increase in value by 2.3% each year. Write a function to model this situation, then find the value of the investment after 15 years.

$f(x) = 5000(1.023)^x$

$f(15) = 5000(1.023)^{15}$

$= \$7032.42$

42. A baseball card that was valued at $200 in 1980 has increased in value by 7% each year. Write a function to model this situation, then find the value of the card in 2016.

$f(x) = 200(1.07)^x$

$f(36) = 200(1.07)^{36}$

$= \$2284.79$

43. Miles invested $2,400 into a retirement account that earns 1.8% interest compounded bimonthly. Write a function to model this situation, then find the balance of the account after 25 years.

$f(x) = 2400\left(1+\frac{.018}{24}\right)^{24x}$

$f(25) = 2400\left(1+\frac{.018}{24}\right)^{24\cdot 25}$

$= \$3763.31$

44. Sarah took out a $30,000 loan at a 4% interest rate to put a new pool in her backyard. If the interest is compounded quarterly, write a function to model this situation. How much will interest will she have paid after 12 years?

$f(x) = 30000\left(1+\frac{.04}{4}\right)^{4x}$

$f(12) = 30000\left(1+\frac{.04}{4}\right)^{4\cdot 12}$

$= 48366.78$

Interest pd = $18,366.78

Topic 8: Regression

45. The table below shows the value of a stock over the course of five years. Using an **exponential model**, write an equation for the curve of best fit, then estimate the value of the stock in 2025.

	Year	Value ($)
0	1998	400
1	1999	438
2	2000	480
3	2001	525
4	2002	575

$f(x) = 400.07 \cdot 1.09^x$ $\quad x=27$

$f(27) = 400.07 \cdot 1.09^{27}$

$= \$4098.75$

46. The table below shows the wind speed along with the corresponding wind chill factor. Using a **logarithmic model**, write an equation for the curve of best fit, then find the approximate wind speed that corresponds to a wind chill factor of -18 °F.

Wind Speed (mi/hr)	Wind Chill Factor (°F)
3	1
5	-3
14	-8
20	-11
27	-14

$f(x) = 7.91 - 6.41 \cdot \ln x$

$-18 = 7.91 - 6.41 \ln x$

$-25.91 = -6.41 \ln x$

$4.04 = \ln x$

$x = e^{4.04}$

$x = 56.95 \approx$ 57 mph

47. The table below shows the number of seniors at Greenville High School that are also taking college credit classes during certain years. Which model would best fit this data: linear, quadratic, or cubic? Use the model to write a best-fit equation, then estimate the number of seniors taking college classes in 2020.

Linear $r = .987$

	Year	Students
0	2004	18
2	2006	24
4	2008	39
8	2012	48
11	2015	64

$f(x) = 4.09x + 18.16$

$f(16) = 4.09(16) + 18.16$

$= 83.6 \approx$ 84 seniors

48. The table below shows the number of used textbooks sold each year on an online bookstore. Which model would best fit this data: cubic, quartic, or exponential? Use the model to write a best-fit equation, then estimate the number of books sold in 2014

Quartic $r^2 = 1$

	Year	Books Sold (thousands)
0	2008	1
1	2009	6.2
2	2010	9.7
3	2011	12.1
4	2012	13.3

$f(x) = -0.03x^4 + 0.28x^3 - 1.47x^2 + 6.43x + 1$

$f(6) = -0.03(6)^4 + 0.28(6)^3 - 1.47(6)^2 + 6.43(6) + 1$

$f(6) = 8.26$

$8,260